The mathematical contribution of Gonzalo Contreras

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Gonzalo Contreras was born in Vila de Mar in Chile. He completed his undergraduate degree at the Catholic University of Chile, his master's degree at Warwick University, and his PhD, under the supervision of Ricardo Mañé, at IMPA in 1990. The topics covered in his thesis are hyperbolic flows, linking numbers, and invariant measures.

Gonzalo passed away this year at 63 years old in Guanajuato, Mexico, and left behind a truly impressive mathematical legacy. He spent most of his career as a professor at CIMAT (Mexico).

He had several collaborators, and all of them have a captivating memory of him, and will remember him as a brilliant mathematician.

During his scientific career, he made important contributions in several areas, mainly in Aubry-Mather Theory, Dynamics of the Geodesic Flows, Hamiltonian Dynamics, Linking Numbers, Lagrangian graphs, Hamilton-Jacobi equations, Partial Differential Equations, Thermodynamic Formalism, and Ergodic Optimization. He was invited to give a lecture at the International Congress of Mathematicians in 2010.

The text "Global minimizers of autonomous Lagrangians", written jointly with R. Iturriaga, became a fundamental reference in Aubry-Mather Theory.

Among his countless works, we highlight some:

1. Proof of the C^2-stability conjecture for geodesic flows of closed surfaces, joint work with M. Mazzucchelli (2024). This work resolves an important conjecture that remained unsolved for several years

2. Genericity of geodesic flows with positive topological entropy on S^2, joint work with G. P. Paternain (2002). This paper shows that geodesic flows

on S^2 having positive topological entropy is an open and dense set in the C^2-topology.

3. Generic hyperbolicity of Aubry sets on surfaces, joint work with A. Figalli and L. Rifford (2015). This paper considers generic properties (in the sense of Mañé) of the structure of the Aubry set in the case of dimension 2; it is proved that C^2-generically the Aubry set is hyperbolic in its energy level.

4. Geodesic flows with positive topological entropy, twist maps, and hyperbolicity, joint work with J-M. Gambaudo, R. Iturriaga and G. P. Paternain (2010). In this work, the authors show that on any closed C^∞-manifold M of dimension $n \geq 2$, the set of C^∞-Riemannian metrics whose geodesic flow on the unit tangent bundle admits a nontrivial hyperbolic basic set is open and dense in the C^2-topology.

5. The Palais-Smale condition and Mañé's critical values, G. Contreras, R. Iturriaga, G. P. Paternain and M. Paternain (2000). The authors show that the critical value can also be characterized by conditions on the action function satisfying the Palais-Smale condition on energy levels.

6. The asymptotic Maslov index and its applications, joint work with J-M Gambaudo, R. Iturriaga and G. P. Paternain (2003). This paper introduces an ergodic version of the notion of Maslov index. The results are applied to the study of Lagrangian and Hamiltonian dynamics.

7. Ground states are generically a periodic orbit (2016.) This work proves the Mañé conjecture the setting of Ergodic Optimization. The paper shows that for expanding maps on a compact metric space, the maximizing measure for generic Lipschitz functions is supported on a periodic orbit.

8. Lyapunov minimizing measures for expanding maps of the circle, joint work with A. O. Lopes and P. Thieullen (2001). One of the first papers in Ergodic Optimization and where it presented a partial proof of the Mañé conjecture

9. Non-hyperbolic surfaces having all ideal triangles with finite area, joint work with R. Ruggiero (1997). The paper shows examples of compact surfaces with negative curvature at all points except along a simple closed geodesic, where the curvature is zero at every point, having the property that all ideal triangles in the universal covering have uniformly bounded area and yet, the geodesic flow is not Anosov. These examples have low differentiability class, up to C^3 , and it was shown later on that if the uniform

boundedness of the area of ideal triangles in the universal covering holds for a C^4 closed surface of nonpositive curvature, then the geodesic flow must be Anosov.

10. The derivatives of equilibrium states (1995). In this work it is presented several results about differentiability related to variations of Holder potentials (and other entities related to the equilibrium states of the potential) in Thermodynamic Formalism.